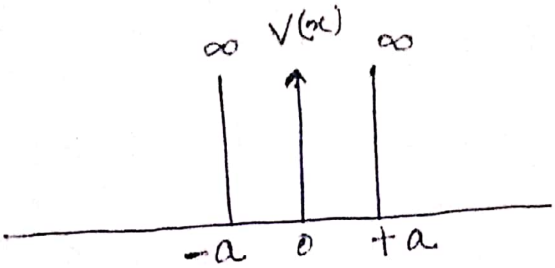
1. One Dimensional infinite Square well



Let us consider a particle of mass 'm' confined in region of width 2a from $x = -a$ to $x = a$. The particle at $x = \pm a$ experiences large force. Therefore, the potential energy

$$V(x) = \begin{cases} 0 & |x| < a \\ \infty & |x| > a \end{cases} \quad -(1)$$

The time independent schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \quad -(2)$$

Since Potential is infinite at $x = \pm a$, the Probability of finding the particle outside the well is zero.

for $|x| > a$

$$\psi(x) = 0 \quad \text{at } x = \pm a \quad -(3)$$

For $|x| < a$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \Rightarrow \frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi \quad -(4)$$

$$\Rightarrow \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \Rightarrow \frac{d^2\psi}{dx^2} + K^2\psi = 0 \quad -(5)$$

$$K = \sqrt{\frac{2mE}{\hbar^2}}$$

The general solution is given by

$$\psi(x) = A\sin Kx + B\cos Kx \quad -(6)$$

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Applying the Boundary condition; $x = a$

$$A \sin ka + B \cos ka = 0 \quad - (7)$$

and at $x = -a$

$$-A \sin ka + B \cos ka = 0 \quad - (8)$$

from Eq (7) & (8)

$$A \sin ka = 0 \ \& \ B \cos ka = 0$$

We cannot allow, $A = B = 0$ (trivial solution)

The two possible solution are

$$A = 0 \ \& \ \cos ka = 0$$

or

$$B = 0 \ \& \ \sin ka = 0$$

$\Rightarrow ka = n\pi/2$

Thus the eigenfunction are

$$\psi_n(x) = B \cos\left(\frac{n\pi x}{2a}\right) \text{ or}$$

$$\psi_n(x) = A \sin\left(\frac{n\pi x}{2a}\right)$$

In order to normalize,

$$\int_{-a}^{a} \psi_n^*(x)\, \psi_n(x)\, dx = 1$$

$$A^2 \int_{-a}^{a} \sin^2\left(\frac{n\pi x}{2a}\right) dx = B^2 \int_{-a}^{a} \cos^2\left(\frac{n\pi x}{2a}\right) dx = 1$$

$$A = B = \frac{1}{\sqrt{a}}$$

$$\psi_n(x) = \frac{1}{\sqrt{a}} \cos\left(\frac{n\pi x}{2a}\right); \quad n = 1, 3, 5 \ldots$$

$$\psi_n(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{n\pi x}{2a}\right); \quad n = 2, 4, 6 \ldots$$

The energy eigen value, $E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{8ma^2}$; $n = 1, 2, 3 \ldots$

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